

Geometry CH4

- Altitude: Height. Any line segment from a vertex to the opposite side of a polygon that is perpendicular to that side
- Congruent triangles: Identical triangles that are the same shape and size. Corresponding sides and angles are equal. The symbol is \cong
- Interval: Part of a line including the endpoints
- Median: A line segment that joins a vertex to the opposite side of a triangle that bisects that side
- Perpendicular: A line that is at right angles to another line. The symbol is \perp
- Polygon: General term for a many sided plane figure. A closed plane (two dimensional) figure with straight sides
- Quadrilateral: A four-sided closed figure such as a square, rectangle, trapezium etc.
- Similar triangles: Triangles that are the same shape but different sizes. The symbol is \sim
- Vertex: The point where three planes meet. The corner of a figure
- Vertically opposite angles: Angles that are formed opposite each other when two lines intersect

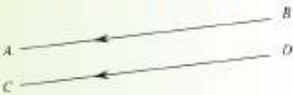
The point is called B.



The interval (part of a line) is called AB or BA.



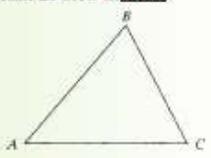
If AB and CD are parallel lines, we write $AB \parallel CD$.



This angle is named $\angle BAC$ or $\angle CAB$. It can sometimes be named $\angle A$.

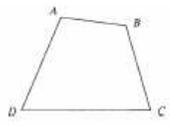


Angles can also be written as \hat{BAC} or $\sphericalangle BAC$.



This triangle is named $\triangle ABC$.

To name a quadrilateral, go around it: for example, BCDA is correct, but ACBD is not.



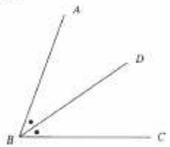
This quadrilateral is called ABCD.

Producing a line is the same as extending it.



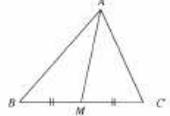
Line AB is produced to C.

$\angle ABD$ and $\angle DBC$ are equal.

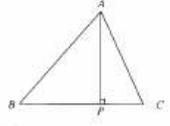


DB bisects $\angle ABC$.

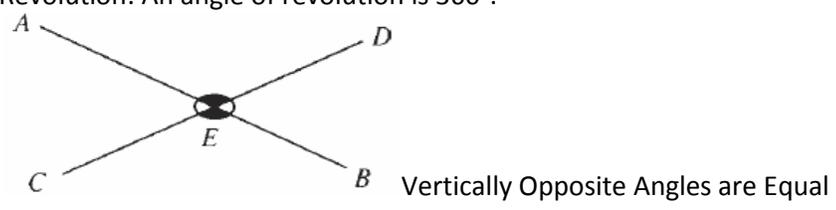
AM is a median of $\triangle ABC$.

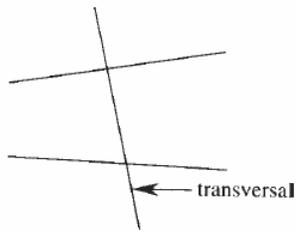


AP is an altitude of $\triangle ABC$.



- Right Angle: A right angle is 90° . (Complementary angles are angles whose sum is 90°)
- Obtuse Angle: $90^\circ < x < 180^\circ$
- Straight Angle: A straight angle is 180° . (Supplementary angles are angles whose sum is 180°)
- Reflex Angle: $180^\circ < x < 360^\circ$
- Revolution: An angle of revolution is 360° .

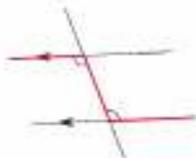




Alternate angles



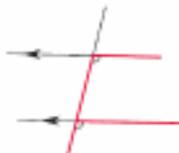
If the lines are parallel, then alternate angles are equal.



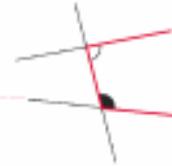
Corresponding angles



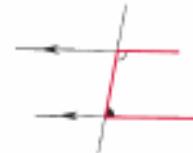
If the lines are parallel, then corresponding angles are equal.



Cointerior angles



If the lines are parallel, cointerior angles are supplementary (i.e. their sum is 180°).

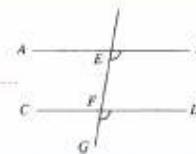


Tests for parallel lines

If alternate angles are equal, then the lines are parallel.



If corresponding angles are equal, then the lines are parallel.



If cointerior angles are supplementary, then the lines are parallel.



If 2 lines are both parallel to a third line, then the 3 lines are parallel to each other. That is, if $AB \parallel CD$ and $EF \parallel CD$, then $AB \parallel EF$.



A scalene triangle has no two sides or angles equal.

A **right** (or right-angled) triangle contains a right angle.
 An **isosceles** triangle has two equal sides.
The angles (called the base angles) opposite the equal sides in an isosceles triangle are equal.
 An **equilateral triangle** has three equal sides and angles.
 An **obtuse-angled** triangle contains an obtuse angle.
 Interior Angle Sum of any triangle is 180°

Exterior angle of a triangle



The exterior angle in any triangle is equal to the sum of the two opposite interior angles. That is,

$$x + y = z$$

Congruency Tests - Two triangles are congruent if

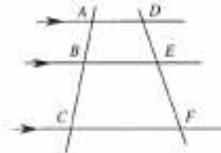
- SSS : all three pairs of corresponding sides are equal
- SAS: two pairs of corresponding sides and their included angles (angle between the 2 sides) are equal
- AAS: two pairs of angles and one pair of corresponding sides are equal
- RHS : both have a right angle, their hypotenuses are equal and one other pair of corresponding sides are equal

Similarity Tests – Two triangle are similar if

- three pairs of corresponding angles are equal (If 2 pairs of angles are equal then the third pair must also be equal)
- three pairs of corresponding sides are in proportion
- two pairs of sides are in proportion and their included angles are equal

Ratio of intercepts

The following result comes from similar triangles.



When two (or more) transversals cut a series of parallel lines, the ratios of their intercepts are equal.

That is, $AB:BC = DE:EF$

or $\frac{AB}{BC} = \frac{DE}{EF}$

$$c^2 = a^2 + b^2$$

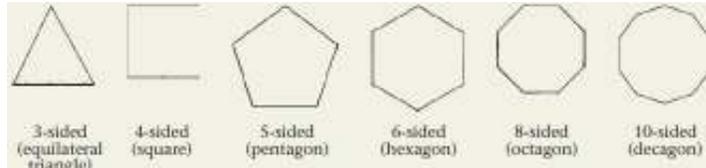
$$c = \sqrt{a^2 + b^2}$$

Pythagoras Theorem If $c^2 = a^2 + b^2$, then $\triangle ABC$ must be right angled

A **quadrilateral** is any four-sided figure (In any quadrilateral the sum of the interior angles is 360°)

A **polygon** is a closed plane figure with straight sides.

A regular polygon has all sides and all interior angles equal.



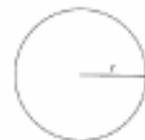
The sum of the interior angles of an n-sided polygon is given by:

$$S = 180n - 360 \text{ or } S = (n - 2) \times 180^\circ$$

The sum of the exterior angles of any polygon is 360°

Area

Circle



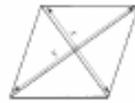
$$A = \pi r^2$$

Rectangle



$$A = lb$$

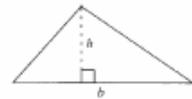
Rhombus



$$A = \frac{1}{2}xy$$

(x and y are lengths of diagonals)

Triangle



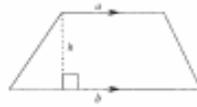
$$A = \frac{1}{2}bh$$

Square



$$A = s^2$$

Trapezium



$$A = \frac{1}{2}h(a + b)$$

Parallelogram



$$A = bh$$

Parallelogram



A parallelogram is a quadrilateral with opposite sides parallel

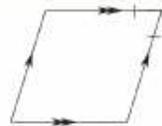
PROPERTIES

- opposite sides of a parallelogram are equal
- opposite angles of a parallelogram are equal
- diagonals in a parallelogram bisect each other
- each diagonal bisects the parallelogram into two congruent triangles

TESTS

- A quadrilateral is a parallelogram if:
- both pairs of opposite sides are equal
 - both pairs of opposite angles are equal
 - one pair of sides is both equal and parallel
 - the diagonals bisect each other

Rhombus



A rhombus is a parallelogram with a pair of adjacent sides equal

PROPERTIES

- the same as for parallelogram, and also
- diagonals bisect at right angles
- diagonals bisect the angles of the rhombus

TESTS

- A quadrilateral is a rhombus if:
- all sides are equal
 - diagonals bisect each other at right angles

Rectangle



A rectangle is a parallelogram with one angle a right angle

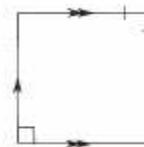
PROPERTIES

- the same as for a parallelogram, and also
- diagonals are equal

TEST

A quadrilateral is a rectangle if its diagonals are equal

Square

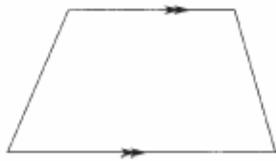


A square is a rectangle with a pair of adjacent sides equal

PROPERTIES

- the same as for rectangle, and also
- diagonals are perpendicular
- diagonals make angles of 45° with the sides

Trapezium



A trapezium is a quadrilateral with one pair of sides parallel

Kite



A kite is a quadrilateral with two pairs of adjacent sides equal