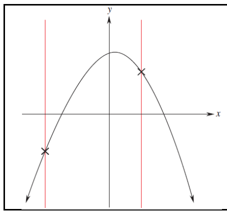
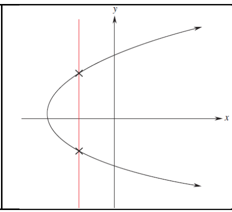
Functions and Graphs

A **function** is a special type of relation. It is like a machine where for every INPUT there is only one OUTPUT.

A **relation** is a set of ordered points (*x* , *y*) where the variables x and y are related according to some rule.

	If a vertical line cuts a graph only once anywhere along the graph, the graph is a function.		If a vertical line cuts a graph in more than one place anywhere along the graph, the graph is not a function.

Function notation

If *y* depends on what value we give *x* in a function, then we can say that *y* is a function of *x* . We can write this as y = *f(x)*. If *y = f (x)* then *f (a)* is the value of *y* at the point on the function where *x = a*

Intercepts

One of the most useful techniques is to find the x- and y -intercepts. For x -intercept, *y = 0* For y -intercept, *x = 0*

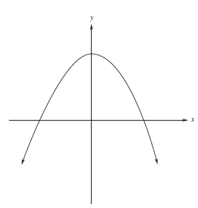
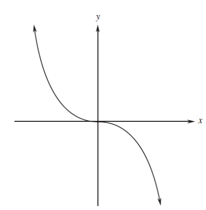
Domain and Range

The *x -coordinate* is called the independent variable and the *y -coordinate* is the dependent variable. The set of all real numbers *x* for which a function is defined is called the domain. The set of real values for *y* or *f(x)* as *x* varies is called the range (or image) of *f* .

Odd and Even functions

If a curve is increasing, as x increases, so does y , and the curve is moving upwards, looking from left to right.

If a curve is decreasing, then as x increases, y decreases and the curve moves downwards from left to right.

	For even functions, *f*(*x*) = *f* (*-x*) for all values of *x* .		Functions are odd if they have point symmetry about the origin. A graph rotated 180° about the origin gives the original graph. For odd functions, f (-x) = -f (x) for all values of x in the domain.

For the family of functions *y* = *k f* (*x*) , as *k* varies, the function changes its slope or steepness.

For the family of functions *y* = *f (x)*+ *k*, as *k* varies, the graph moves up or down (vertical translation).

For the family of functions *y = f (x + k)* , as *k* varies, the graph moves left or right (horizontal translation).

Linear Functions (straight lines)

Gradient form: *y* = *mx* + *b* has gradient *m* and *y* -intercept *b*

General form: *ax* + *by* + *c* = 0

The linear function *ax* + *by* + *c* = 0 has domain {all real *x* } and range {all real *y* } where *a* and *b* are non-zero.

x = a is a vertical line with x -intercept a	Domain: {x: x = a}	Range: {all real y }
y = b is a horizontal line with y -intercept b	Domain: {all real x }	Range: {y: y = b}

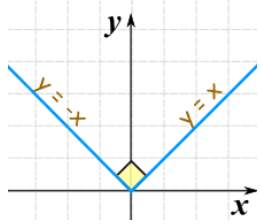
Quadratic Functions

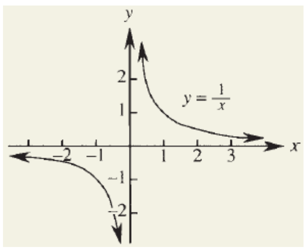
$f(x) = ax^2 + bx + c$ is the general equation of a parabola. If a a>0 the parabola is concave upwards (U shaped). If a a<0 the parabola is concave downwards (^ shaped).

Absolute Value Functions

Use table of values e.g. -3 ≤ x ≤ 3 or definition of absolute value.

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$





Hyperbola

Equation: $xy = a$ or $y = \frac{a}{x}$

x cannot be = 0 (undefined)

The function $f(x) = \frac{a}{bx + c}$ is a hyperbola with domain $\{\text{all real } x\text{: } x \neq -\frac{c}{b}\}$ and range $\{\text{all real } y\text{: } y \neq 0\}$

Circles

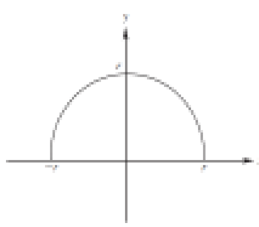
A graph whose equation is in the form $x^2 + ax + y^2\ by + c = 0$ has the shape of a circle.

There is a special case of this formula: The graph of $x^2 + y^2 = r^2$ is a circle, centre (0, 0) and radius r

The circle x2 + y2 = r2 has domain: {x: -r ≤ x ≤ r} and range: {x: -r ≤ y ≤ r}

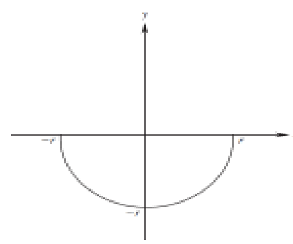
The equation of a circle, centre (a , b) and radius r is $(x - a)^2\ (y - b)^2 = r^2$

$y = \sqrt{r^2 - x^2}$ is the semi-circle above the x-axis since its range is $y \geq 0$ for all values.



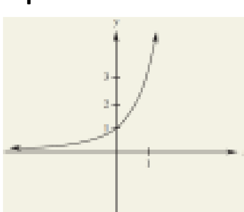
The domain is $\{x: -r \leq x \leq r\}$ and the range is $\{y: 0 \leq y \leq r\}$

$y = -\sqrt{r^2 - x^2}$ is the semi-circle **below** the x-axis since its range is $y \leq 0$ for all values.



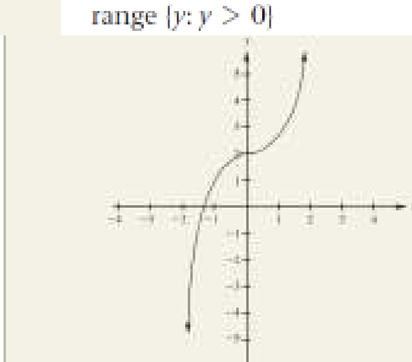
The domain is $\{x: -r \leq x \leq r\}$ and the range is $\{y: -r \leq y \leq 0\}$

Exponentials



From the graph, *x* can be any real value (the equation shows this as well since any *x* value substituted into the equation will give a value for *y*). From the graph, *y* is always positive, which can be confi rmed by substituting different values of *x* into the equation. Domain: all real *x* Range: y>0

The exponential function $y = a^x$ has domain $\{\text{all real } x\}$ and range $\{y: y > 0\}$



The function can have any real x or y value:
Domain: {all real x}
Range: {all real y}

Cubic Function

A cubic function has an equation where the highest power of x is x^3.

When finding the domain, we look for values of x that are impossible. For example, with the hyperbola you have already seen that the denominator of a fraction cannot be zero. For the range, we look for the results when different values of x are substituted into the equation. For example, x^2 will always be zero or a positive number.

Limits

The exponential function and the hyperbola are examples of functions that approach a limit. The curve $y = a^x$ approaches the x-axis when x approaches very large negative numbers, but never touches it.

That is, when $x \to -\infty$, $a^x \to 0$.

Putting $a^{-\infty}$ into index form gives

$$a^{-\infty} = \frac{1}{a^{\infty}} = \frac{1}{\infty} \doteqdot 0$$

A line that a graph approaches but never touches is called an ***asymptote****.*

We say that the limit of a^x as x approaches $-\infty$ is 0. In symbols, we write $\lim_{x \to -\infty} a^x = 0$.

Continuity

Many functions are continuous. That is, they have a smooth, unbroken curve (or line). However, there are some discontinuous functions that have gaps in their graphs. The hyperbola is an example. If a curve is discontinuous at a certain point, we can use limits to find the value that the curve approaches at that point.

Regions

Regions can be bounded or unbounded.

A bounded region means that the line or curve is included in the region.

E.g. Remember that x = 3 is a vertical line with x -intercept 3.

An unbounded region means that the line or curve is not included in the region.

For lines that are not horizontal or vertical, or for curves, we need to check a point to see if it lies in the region.

Sometimes a region includes two or more inequalities. When this happens, sketch each region on the number plane, and the final region is where they overlap (intersect).

The first quadrant is where x and y values are both positive.