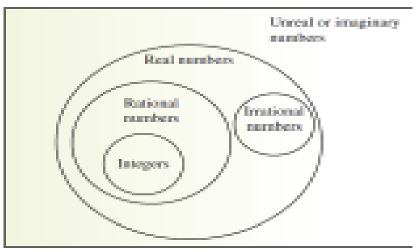
Basic Algebra (Chapter 1)

Definitions

- Absolute value: The distance of a number from zero on the number line. Hence it is the magnitude or value of a number without the sign.
- Directed numbers: The set of integers or whole numbers... -3, -2, -1, 0, 1, 2, 3...
- Exponent: Power or index of a number. For example 2^3 has a base number of 2 and an exponent of 3
- Index: The power of a base number showing how many times this number is multiplied by itself e.g. 2^3=2x2x2 The index is 3
- Indices: More than one index (plural)
- Recurring decimal: A repeating decimal that does not terminate e.g. 0.777777 … is a recurring decimal that can be written as a fraction. More than one digit can recur e.g. 0.14141414...
- Scientific notation: Sometimes called standard notation. A standard form to write very large or very small numbers as a product of a number between 1 and 10 and a power of 10 e.g. 765 000 000 is 7.65×10^8 in scientific notation.



Integers are whole numbers that may be positive, negative or zero. E.g. -4, 7, 0, -11
Rational numbers can be written in the form of a fraction $\frac{a}{b}$ where a and b are integers, $b \neq 0$. e.g. $1\frac{3}{4}, 3.7, 0.\dot{5}, -5$
Irrational numbers cannot be written in the form of a fraction $\frac{a}{b}$ (that is, they are not rational) e.g. $\sqrt{2}$, π

Same signs = +
\+ + = +
− − = +
Different signs = −
\+ − = −
− + = −

Order of Operations

1. Brackets 2. Multiply or Divide 3. Add or Subtract

Rounding off: To round a number off to a certain number of decimal places, look at the next digit to the right. If it's 5 or higher, round it up otherwise for 4 or less round it down.

Index Laws

$x^m x^n = x^{m+n}$	$(xy)^m = x^m y^m$
$\frac{x^m}{x^n} = x^{m-n}$	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
$(x^m)^n = x^{mn}$	

$x^0 = 1$

$$x^{-n} = \frac{1}{x^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Scientific Notation

A number in scientific notation is written as a number between 1 and 10 multiplied by a power of 10.

Significant Figures

The concept of significant figures is related to rounding off. When we look at very large (or very small) numbers, some of the smaller digits are not significant. Even though zeros may not be significant, they are still necessary. Scientific notation uses the significant figures in a number.

Absolute Values

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

We write the absolute value of x as |x|

$$|ab| = |a| \times |b| \qquad |-a| = |a|$$

$$|a|^2 = a^2 \qquad |a-b| = |b-a|$$

$$\sqrt{a^2} = |a| \qquad |a+b| \leq |a| + |b|$$

Algebra and Surds (Chapter 2)

Definitions

- Binomial: A mathematical expression consisting of two terms such as x + 3 or 3x – 1
- Binomial product: The product of two binomial expressions such as (x + 3) (2x - 4)
- Expression: A mathematical statement involving numbers, pronumerals and symbols e.g. 2x – 3
- Factorise: The process of writing an expression as a product of its factors. It is the reverse operation of expanding brackets i.e. take out the highest common factor in an expression and place the rest in brackets e.g. 2y - 8 = 2(y - 4)
- Pronumeral: A letter or symbol that stands for a number
- Rationalising the denominator: A process for replacing a surd in the denominator by a rational number without altering its value
- Surd: From 'absurd'. The root of a number that has an irrational value e.g. 3. It cannot be expressed as a rational number
- Term: An element of an expression containing pronumerals and/or numbers separated by an operation such as + - * / e.g. 2x, -3
- Trinomial: An expression with three terms such as $3x^2$ -2x +1

Distributive Law: **a (b + c) = ab + ac**

A **binomial expression** consists of two numbers, for example x + 3. A set of two binomial expressions multiplied together is called a binomial product. Example: (x+3) (x-3) Each term in the first bracket is multiplied by each term in the second bracket. $(a+b)(x+y) = ax + ay + bx + by$ $\quad (a+b)(x+y+z) = ax + ay + az + bx + by + bz$

Difference of 2 Squares: $(a+b)(a-b) = a^2 - b^2$

Perfect Square: $(a+b)^2 = a^2 + 2ab + b^2 \quad (a-b)^2 = a^2 - 2ab + b^2$

To factorise an expression, we use the **distributive law: ax + bx = x (a+b)**

$$ax + bx + ay + by = x(a+b) + y(a+b)$$
$$= (a+b)(x+y)$$

Grouping in Pairs:

A trinomial is an expression with three terms, for example $x^2 - 4x + 3$ Factorising a trinomial usually gives a binomial product. $x^2 + (a+b)x + ab = (x+a)(x+b)$

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Perfect Squares:

Difference of 2 cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

To complete the square on $a^2 + pa$, divide p by 2 and square it.

$$a^2 + pa + \left(\frac{p}{2}\right)^2 = \left(a + \frac{p}{2}\right)^2$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$
$$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
$$(\sqrt{x})^2 = \sqrt{x^2} = x$$

Surds

An *irrational number* is a number that cannot be written as a ratio or fraction (rational). Surds are special types of irrational numbers, such as √2, √3 and √5. Some surds give rational values: for example, √9 = 3. Others, like √2, do not have an exact decimal value.

Addition and subtraction of Surds

Calculations with surds are similar to calculations in algebra. We can only add or subtract 'like terms' with algebraic expressions. This is the same with surds.

Multiplying and Dividing with Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$$

Expanding brackets with Surds

The same rules for expanding brackets and binomial products that you use in algebra also apply to surds. Simplifying surds by removing grouping symbols uses these general rules. $\sqrt{a}(\sqrt{b} + \sqrt{c}) = \sqrt{ab} + \sqrt{ac}$

Binomial product:

$$(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$$

Perfect squares:

$$(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b \qquad (\sqrt{a} - \sqrt{b})^2 = a - 2\sqrt{ab} + b$$

Difference of two squares:

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

Rationalising the denominator

Rationalising the denominator of a fractional surd means writing it with a rational number (not a surd) in the denominator. For example, after rationalising the denominator, $\frac{3}{\sqrt{5}}$ becomes $\frac{3\sqrt{5}}{5}$.

Squaring a surd in the denominator will rationalise it since $(\sqrt{x})^2 = x$.

$$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

When there is a binomial denominator, we use the difference of two squares to rationalise it, as the result is always a rational number.

To rationalise the denominator of $\frac{\sqrt{a} + \sqrt{b}}{\sqrt{c} + \sqrt{d}}$, multiply by $\frac{\sqrt{c} - \sqrt{d}}{\sqrt{c} - \sqrt{d}}$

Completing the Square

To complete the square on $a^2 + pa$, divide p by 2 and square it.

$$a^2 + pa + \left(\frac{p}{2}\right)^2 = \left(a + \frac{p}{2}\right)^2$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$